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Mathematics Portfolio  
CRIN E07  
December 7, 2010

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## **Overview of Methods Class and Practicum Experiences**

To me, a portfolio is an opportunity to review academic work and classroom experiences and synthesize these elements into a cohesive framework. In the following pages, I have compiled a comprehensive overview of my classroom observations and methods class work and then reflect on how these two strands of experience will inform my future mathematics instruction.

The greatest area of overlap between my two areas of math experience has been in error analysis. My practicum teacher administers a pre-assessment at the beginning of every unit of study and I have been asked to grade and group the pre-assessments for the money and rounding units. Analyzing the source of student error has proven instrumental in organizing these students into three small groups for rotation among math centers.

In my placement, the teachers work as a team and I follow my cooperating teacher's class to another room for math. I observe a math teacher who has taught for five years and is in her first year teaching 3<sup>rd</sup> grade after three years in kindergarten and two years in 1<sup>st</sup> grade. She has support for this new curriculum since all 3<sup>rd</sup> grade teachers at Matoaka use the same homework, in-class materials, and lesson plans and share identical end of unit tests. Lessons taught to date are place value, regrouping, adding and subtracting three-digit numbers, money, inverse relationships and four fact families, and rounding.

## Taught Problems and Learning Plan (with Critiques)

### Problem #1

#### **Thoughts:**

*In this portfolio, I have included the instruction actually taught in the classroom to highlight the modifications I made on the spot as well as those changes I intend to make in the future.*

*Narcissism is a powerful instructional tool particularly in the earlier grades. Students are innately curious about their names and this lesson capitalizes on this interest. I also like the connection with language arts and the students need to differentiate between consonants and vowels.*

**Topics:** Computation and Estimation (SOL 3.8, 3.9) and Probability and Statistics (SOL 3.21, 3.22)

**Expected Heuristics:** Choose the operation

**Appropriate Grade:** Grade 3

**Problem:** *What's in a name?* Discover how much your first name is worth if every consonant has a value of 5 cents and vowels are 10 cents. Compare the value of your name with those of your friends. Graph the class results to see which names have the greatest and least value. Find the range of numbers (the difference between the greatest and least value). What do you notice on the graph about the names with the greatest and least value?

**Context of lesson:** I taught this lesson as part of our three-group rotation in math. The class is comprised of twenty-three students who are all proficient English speakers and none of whom have an IEP. I worked at the “seatwork center” with seven or eight students at a time. I integrated this activity into one of our early lessons on money and changed my original plan from dollar values to cents to give the students practice recognizing and calculating nickels and dimes. Coin recognition has proven to be more problematic for the class than dollar recognition. I provided play money to support the students’ calculations and sticky dots (used to identify files) for use in constructing graphs. Each dot was valued at five cents (one yellow for a nickel, two reds for a dime) and students created vertical towers on strips of graph paper to represent the “value” of their name. The students’ names were written below the tower of dots. I closed the math class by displaying the completed chart. I was not present the following day but understand that the teacher led a class discussion about the range of values.

**Students’ thinking about the problem:** Students demonstrated strong interest in breaking down their names into component parts. The activity engendered strong discussion about what letters were vowels and what were consonants and I was somewhat taken aback that not all students had mastered this grammar. The “struggling group” made steady and productive use of the play coins for support while two of the students in the middle ability group used the coins. Twice, a student helped a classmate in assembling the dot tower noting that reds had to be represented in even numbers. None of the students in the high ability group used the manipulatives and because they

finished so quickly, I asked them to repeat the exercise using their last names and then express the total value of both names in dollars and cents. As each group moved through the station, the students placed their names in value order on a sheet of paper, rearranging as needed. When finished, I taped these towers onto the paper for display.

**Modifications to future instruction:** I liked connecting the lesson to coins rather than dollars and will keep this variation. I need to add difficulty for the higher-ability students and therefore when using the coin model, I might have a quarter represent uppercase letters and would definitely include first and last names. I would prefer to introduce this activity to the whole group and ask students to reflect on the names in the classroom. Students would then make a prediction about what names would have the least and greatest values and specify the specific values for these names. At the end of the class, when the graph is displayed for the class, the closest predictions would be revealed. I also envision that this activity could be used to introduce the statistical concept of mode since the recurrence of a number would be easily apparent. I believe that median and mean are above the current ability level of this group of students.

**What I learned from interactions with students:** As my first taught lesson, I learned that many students were reluctant to take any action on their own and that I needed to encourage students without actually doing their work. Some students needed manipulatives as support for the problem. Some were still unclear about the shape, size, and appearance of dimes and nickels and having the play money to reinforce these attributes was extremely useful. For this activity, students worked independently on their own names but they were naturally drawn to comment upon each other's work and offer support as necessary. Some students in the struggling group showed some difficulty in counting by 5s thus reinforcing my concerns that fundamental skills are shaky and need to be briefly drilled as part of every class.

Weekly Problem: Elementary School (2008). *Math by the Month, Teaching Children Mathematics*. Retrieved September 19, 2010, from <http://www.nctm.org/resources/archive.aspx?id=3604&journalid=4>

**Problem #2****Thoughts:**

*I wrote this particular lesson plan as a means to integrate literature into our quiet and rather flat classroom. This book is funny and engages the students immediately with a lilting rhyme and absurd concept. I love to read aloud and believe that this activity builds solidarity and interest in a class. This specific rhyme presents a wealth of potential for math problem-solving and computational practice.*

**Topics**

Computation and Estimation (SOL 3.8,3.10) and Number and Number Sense (3.3)

**Expected Heuristics**

- Make an organized list.
- Make a table.
- Choose the operation.

**Appropriate Grades**

Grade 3

**Problem**

- **“One Hungry Monster” Food Analysis:** Examine the kinds of food the boy gave to the hungry monsters. Using the following grocery advertisement, determine if the total cost of the fruit and vegetables was more or less expensive than the cost of the other items served. What is the cost difference?
- **Grocery Store advertisement:**
  - eggplant 50 cents
  - pickled pear 30 cents
  - pumpkin \$3.00
  - watermelon \$2.00
  - bread \$2.50
  - spaghetti dinner \$2.00
  - roasted turkeys \$4.00
  - frozen pizza \$5.00
  - peanut butter \$1.00
  - apple juice \$1.50

**Context of lesson:** Again, I taught this lesson as part of our classroom’s three-group rotation. I read the brief rhyming story to the students and then provided two additional copies of the book for reference. Given the fact that the students needed to share a book, I asked students to work together to differentiate the food items and to determine the specific quantities and costs of each item. The students searched through the books for information and discussed questions such as whether peanut butter is a vegetable and whether apple juice can be considered a fruit. While these might not be mathematical questions, they prompted thoughtful discussions and some detailed debate. Students wrote down their calculations and totals and compared findings at the end of our sessions.

**Students' thinking about the problem:** Students appeared to enjoy the connection with the text, particularly such a rhythmic and humorous book. The excitement of discovery and parsing the answer seemed to dispel general qualms about not receiving explicit instructions from an adult. I did hear a few instances of "I need help" but classmates quickly jumped in and I was not asked to walk students through the problem. Students in all three ability groups were engaged and motivated.

**Modifications to future instruction:** I love the class camaraderie created in a group read-aloud and believe this activity would have been more streamlined and packed more punch as a whole group activity. The entire arc of the lesson does not require a full class session. It could be used as a warm-up activity and is appropriate as a means to teach and nurture problem solving. I would have liked to procure more copies of the book for a small group session. As a whole group lesson, I would transcribe the book on a single double-sided sheet of paper and distribute to each student as a reference.

**What I learned from interactions with students:** I have not seen literature incorporated into math this year and have sparse memories of my children's math classes including books. I was intrigued by the different mood created when using a book as the basis for a math lesson. Students who are often confused or lack confidence about math seemed to be more relaxed and open to the idea of working beyond the bounds of explicit adult direction. The book served to blur lines between content areas and to shake up the established routine of our math instruction. Based on this experience, I would definitely like to fold authentic non-mathematical content (e.g., literature, science, engineering, social studies, health) into math instruction. I believe this blending of content might serve to engage reluctant students and to reinforce learning for those who are already motivated.

O'Keefe, S.H. (1989). *One hungry monster: A counting book in rhyme*. Boston: Little, Brown and Company.

### **Problem #3**

#### **Thoughts:**

*Having purchased “Learning through Problems” after our early lessons on problem solving, I was interested in trying out some of the book’s many wonderful ideas in the classroom. This pizza problem holds intrinsic appeal to our pizza-loving students and ideally I would love to plan it in tandem with “Pizza Counting” by Christina Dobson and “Pizza at Sally’s” by Monica Wellington. In my former school, our pre-kindergarten class had a month-long January cooking unit that involved days of pizza making, these books (and others), and a culminating surprise visit from a pizza deliverer bringing lunch. I would love to recreate that excitement in an elementary setting and this problem provides the perfect staging for such an event.*

#### **Topics**

Number and Number Sense (SOL 3.3) & Computation and Estimation (SOL 3.8)

#### **Expected Heuristics**

Choose the operation, Draw a picture; Guess and check

#### **Appropriate Grades**

Grade 3

#### **Problem**

- **The Pizza Problem:** The class will be having a pizza party and every student will have 2 slices of pizza to eat. The pizzeria has large pizzas with 10 slices each that cost \$13, medium pizzas with 8 slices each that cost \$9, and small pizzas with 6 slices each that cost \$7. How many total slices will we need to order? What pizza order should we place to minimize the number of leftover slices? What pizza order will cost the least and feed the entire class?

**Context of lesson:** I taught this lesson during my classroom’s recent unit on money. Using the document camera, I presented the problem to the whole class. I then solicited student input and drew the different sized pizzas and their number of slices and cost. After distributing a handout detailing the problem to each student, I asked the class to work independently on the problem during the seatwork center of our three-group rotation. The problem was achievable within the allotted 10-15 minutes center. When students had finished their calculations and made proposals, I asked them to compare their conclusions with a partner and discuss any differences.

**Students’ thinking about the problem:** Students were definitely engaged with the idea of organizing a pizza party. The students realized that they were constructing a hypothetical order list but, nonetheless, I began to feel slightly guilty about not actually ordering pizzas at the end of class. Most students were very careful about counting the total number of students in the class and others wondered about including the adults in the total count; I offered no direction on either count but encouraged them to explain who they were including. Many students worked through multiple permutations of the order and when comparing options often added another calculation (usually to include the two classroom adults in their order).

**Modifications to future instruction:** In hindsight, I would definitely have emphasized that students would be calculating a variety of options and that all work might prove valuable. I would actively discourage students from erasing any calculations. In fact, erasing could actually be counterproductive; I found that some students were recreating earlier work when they discovered that an erased option was of use. Of course, the most exciting change would be to end this lesson with an actual pizza party.

**What I learned from interactions with students:** I discovered the benefit of partnering individual work with peer conferencing and review. Students had valuable insights when justifying their conclusions and realized that multiple permutations existed for the same group order. Many students believe that only one answer exists for a given question and I believe this problem confounded this set interpretation and might foster some student flexibility and confidence when tackling a problem.

Trafton, P.R. & Thiessen, D. (1999). *Learning through problems: Number sense and computational strategies*. Portsmouth, N.H.: Heinemann.

### Taught Learning Plan with Critique

#### **Thoughts:**

*When I first asked what lesson I should teach for this methods class, my cooperating teachers simultaneously cried out “inverse relationships” and then burst out laughing. As they both predicted, inverse relationships have proven to be an elusive concept for our third graders. I believe the difficulty is partially attributable to the students’ weak basic math skills. They must focus great attention on the computational challenges of the math and therefore are unable to devote full attention to mastering the concept of inverse relationships and four fact families. We have moved beyond this unit but I believe at least one-third of the class still demonstrates limited proficiency in this topic.*

**Matoaka Elementary School**  
**3<sup>rd</sup> Grade Math Lesson Plan – Inverse Relationships**  
**Christine Ammirati, CRIN E07**

**Lesson Title:** Inverse relationships between addition and subtraction.

**Context/Grade Level:** This lesson is designed for an autumn semester Matoaka Elementary 3<sup>rd</sup> grade class with no IEPs and high English proficiency. The class manifests a range of confidence and abilities in mathematics. This lesson is the first 3<sup>rd</sup> grade lesson to introduce inverse relationships and “four fact families.” This material is a review of 2<sup>nd</sup> grade knowledge and will serve as a foundation for exploring inverse relationships with three digit numbers.

**Objective:** Given two addends, the student will list the four number sentences that constitute a four-fact family.

**SOL Strand:** Number and Number Sense

**SOL:** 3.4: The student will recognize and use the inverse relationships between addition/subtraction ~~and multiplication/division~~ to complete basic fact sentences. Students will use these relationships to solve problems such as  $5 + 3 = 8$  and  $8 - 3 = \underline{\quad}$

**Materials/Resources:**

- Twelve pairs of dice
- Twelve sets of twelve colored tokens (six blue, six red)
- Colored markers
- Index cards

**Approximate time required:** 45 minutes / 1 class period

**Content and Instructional Strategies:**

1. Introduce the relatedness of addition and subtraction by rolling a pair of dice and recording the two number results on a triangle drawn on the whiteboard. With student input and contribution, label each point on the triangle base with the two addends obtained from the dice; write the sum of these two numbers at the apex of the triangle. For visual support, draw blue dots below the triangle equaling the number rolled on the first die and red dots equaling the number rolled on the second die; the dots should be arrayed on a straight line as if on a number line.
2. On the whiteboard next to the triangle, write the number sentence obtained from the dice roll. Ask students to use the commutative property of addition to restate the number sentence and record this new number sentence (with the order of addends reversed) below the original number sentence. Finally, solicit student input on the two related subtraction number sentences that use these three numbers. Emphasize that subtraction is the opposite (or inverse) property of addition; subtraction undoes the work of addition. Write these two subtraction number sentences below the addition equations.
3. Number each of the four number sentences one to four, circle them in one large group, and explain and emphasize the relationship between them using the phrase “four fact family.”
4. Repeat the above modeling with a second roll of the dice. Again, complete the steps in the following order:
  - a. draw the triangle
  - b. record the addends at the base of the triangle
  - c. draw the colored dots representing each die’s number
  - d. calculate the sum of the addends and write at the apex of the triangle
  - e. write the four number sentences of the four fact family as identified by the students
5. Divide the students into pairs. Each pair receives a pair of dice, a set of colored tokens, matching colored markers, and six index cards.
6. Practice: Tell the students that they will be recording “four fact families” for use in the classroom. Encourage them to use the tokens for support. Ask them to work together using the teacher-modeled technique and should roll the dice at least four times. Each roll of the dice should be used to create on an index card, a:
  - a. Labeled triangle
  - b. Colored dots

- c. Four number sentences
7. In ten minutes, after groups have completed at least four index cards, teacher leads a whole group discussion of results. Each student pair should have opportunity to share one index card. Teacher will record these four fact families on the whiteboard.
8. Teacher will then model this same technique for a sum between 12 and 20.
9. Students will then complete two worksheets for individual practice.
10. Summarize the lesson by asking:
  - a. How are addition and subtraction related?
  - b. How can you use addition to check the answer to a subtraction?

**Evaluation/Assessment:**

Teacher places two numbers greater than 20 at the base of a triangle drawn on the whiteboard. On an exit card (index card), students are asked to complete the triangle and to write the associated four number sentences.

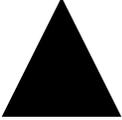
**Differentiation and Adaptations:**

Special education students can be paired with general education students for support. Specialty dice with higher numbers can be used by students seeking / needing greater challenge.

## “Four Fact Family” Triangles

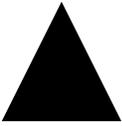
Write the four number sentences for each fact family triangle.

**Example:**

<p style="text-align: center;"><b>3</b></p>  <p><b>1</b>            <b>2</b></p> <p style="text-align: center;"><math>1 + 2 = 3</math> <math>2 + 1 = 3</math> <math>3 - 2 = 1</math> <math>3 - 1 = 2</math></p>	<p style="text-align: center;"><b>8</b></p>  <p><b>3</b>            <b>5</b></p>
<p style="text-align: center;"><b>7</b></p>  <p><b>4</b>            <b>3</b></p>	<p style="text-align: center;"><b>6</b></p>  <p><b>4</b>            <b>2</b></p>
<p style="text-align: center;"><b>11</b></p>  <p><b>5</b>            <b>6</b></p>	<p style="text-align: center;"><b>13</b></p>  <p><b>8</b>            <b>5</b></p>
<p style="text-align: center;"><b>10</b></p>  <p><b>9</b>            <b>1</b></p>	<p style="text-align: center;"><b>15</b></p>  <p><b>7</b>            <b>8</b></p>

Complete the fact family triangle and write the four number sentences.

**Example:**

<p>4</p>  <p>3      ?</p> <p><math>3 + 1 = 4</math> <math>1 + 3 = 4</math> <math>4 - 3 = 1</math> <math>4 - 1 = 3</math></p>	<p>9</p>  <p>6      ?</p>
<p>12</p>  <p>?      7</p>	<p>5</p>  <p>?      4</p>
<p>16</p>  <p>9      ?</p>	<p>?</p>  <p>11      3</p>
<p>19</p>  <p>?      12</p>	<p>?</p>  <p>8      9</p>

## Critique of Taught Lesson Inverse relationships between addition and subtraction.

### **Description of what happened:**

Due to scheduling issues, this learning plan became the third rather than the first lesson on inverse relationships of the unit. The students had already completed a pre-test to assess their knowledge of the “four fact families.” A wide discrepancy in ability and knowledge existed among the twenty-three students in the class. Seven students scored perfect scores on the pre-assessment while eight students scored below 50%. The math teacher had divided the students into three groups based on their performance on the pre-test. By the time I taught my lesson, the students had already received an hour and a half of instruction on inverse relationships, math mountains, and four fact families. To reflect classroom language, I used the term “math mountain” in describing the triangles used in my lesson plan. The lesson’s structure followed our class’ routine three-center rotation with three groups of students moving between seatwork exercises, IXL Math computer activities, and small group instruction with the teacher.

I taught this lesson on my first day working as a substitute in my practicum placement. The students noted the novelty of the situation and were initially chatty and animated but soon settled down. The dice were instrumental in changing the mood of the class and gaining the students’ focus.

I began the lesson with whole group instruction. I used the document camera to project a small whiteboard onto the large screen at the front of the room. Although I had originally planned to use the large classroom whiteboard, I did not want to erase detailed notes the teacher had left from another lesson. In addition, the students are accustomed to the document camera for math instruction and its use became an asset, allowing all students to be able to see the rolls of the dice. Twice, I rolled the dice and asked for student input to construct the “math mountain” with the addends designated by the dice numbers. The students dictated where to place the addends (at the base of the math mountain) and solved for the sum that was then placed at the apex of the mountain. I then asked students to state the four fact family number sentences associated with the two rolled addends. I wrote these number sentences alongside the math mountain and the dice.

After these two initial demonstrations, I divided the students into cooperative pairs. Given the large discrepancy in ability levels, I paired the students within their existing ability groups. For the **struggling students**, I gave them dice and handout #1 and instructed them to go to the small group instruction area to begin to work through the problems with their partner; I joined them about three minutes later. I then gave the students in the **high-skill group** 12-sided dice for their seatwork and placed 24-sided dice on the document camera table for use if greater challenge was desired. Due to the odd number of students in this group, one student worked independently. Because the handouts I prepared were intended as a concept introduction and addressed single-digit numbers, I did not ask these students to use the handout but tasked them with rolling their dice 10 times and creating “math mountains” and four fact families list for each of these 10 rolls. They wrote their math mountains and four fact families in their math practice notebooks. I then met briefly with the **middle group**, directing them to be first rotation at the computers and explaining that they would work with dice when they met with me in their small group. Working individually, they were to work on third grade section C.4: “complete the addition sentence – up to three digits,” (<http://www.ixl.com/math/practice/grade-3-complete-the-addition-sentence-up-to-three-digits>). Using these IXL-generated exercises, I asked them to

write down the problem and its solution in math mountain format in their math practice books. In addition, they were directed to write the four fact families associated with these numbers.

### **Critique and Suggested Modifications for Future Use:**

#### Positives:

- I was relieved and pleased that I had set aside 12-sided and 24-sided dice in advance of the lesson and therefore was able to offer additional challenge for the higher group.
- I was also happy to work within the three-part rotation structure for the different math ability groupings. I felt that this particular lesson benefited enormously from the rotational structure because some students were struggling with the lesson due to weak skills in single-digit addition facts while other students had already mastered the math mountain concept and were ready to apply their knowledge to three digit addition and subtraction challenges.
- Manipulatives were not an original part of my learning plan. When I saw the difficulty my struggling students had with the addition of single digit numbers, I was grateful to have manipulatives readily at hand in the classroom and to be able to integrate them smoothly. I believe the manipulatives increased comprehension for at least four of the eight students in the struggling group.
- The use of math practice notebooks is an effective supplement to computer work. The students can refer back to problems (with their accompanying solutions) long after they have moved forward in their virtual instruction.

#### Negatives:

- In hindsight, I would have immediately split away the high level group to begin work on their own. They did not need my initial introduction and modeling of four fact families and could have used these minutes more productively. In the future, I would prepare separate engagement activities for two discrete ability levels.
- I had not prepared a higher challenge handout for the higher ability group.
- I am accustomed to paper and pencil and chart summaries and therefore favor those methods over working on whiteboards. Whiteboard work is ephemeral and cannot be used as a reference by either the students (for support) or the teacher (to show progress). A step-by-step model displayed on charts affixed to the classroom walls or contained in a math notebook might prove to be effective scaffolding for students.

### **Reflections:**

I felt that this lesson went extremely well and that the sudden departure of the teacher to deal with a family emergency did not derail the instruction. I benefited from the students' comfort with the rotation routine and gained greater appreciation for this structure's effectiveness in differentiating between ability groups and offering varied approaches on the same concept.

### **Summary of What I Learned:**

- Technology can be seamlessly integrated into classroom management and learning.
- Small group rotation improves classroom management and differentiates instruction.
- Need to consider two separate lesson plans to address widely different ability levels.
- Manipulatives must be ready to integrate into all lessons as needed.
- Independent thinking and confidence in math must be nurtured. I am unclear about how to effect this objective.
- Whiteboard and document camera work does not provide permanent record of methodology or strategies for classroom or student use.
- Student knowledge of basic facts is uneven.

### Learning Plan (still to be taught)

#### **Thoughts:**

*I will be teaching this lesson next week and will partner it with a Language Arts assignment on writing "friendly letters." After completing their calculations, the students will write the principal, enclose their proposed timetable, and advocate for their particular schedules.*

*I have modified this lesson from its original form to include duration in the Objective.*

**Lesson Title:** A Shorter Day

**Context/Grade Level:** This lesson is designed for a midyear Matoaka Elementary 3<sup>rd</sup> grade class with no IEPs and high English proficiency. The class manifests a range of confidence and abilities in mathematics. This lesson follows multiple classes that reviewed components of analog clocks and telling time to the nearest five-minute interval. This lesson is the first of the school year to apply student knowledge of time intervals to a specific context.

**Objective:** Given a modified classroom schedule when the school day is reduced by an hour, the student will calculate a new daily schedule, draw analog clocks representing the new start and end times for each class, and determine the new duration of classes.

**SOL Strand:** Measurement, Number and Number Sense, Computation and Estimation

**SOL:** 3.15. The student will tell time to the nearest five-minute interval and to the nearest minute, using analog and digital clocks.

3.16. The student will identify equivalent periods of time, including relationships among ~~days, months, and years,~~ as well as minutes and hours.

#### **Materials/Resources:**

- Twenty-four analog clock models
- Reference Sheet: Telling Time With an Analog Clock (see attached)
- Handout: Summary: New Timetable for Classes (see attached)
- Handout: Clock Faces for New Class Schedule (see attached)

**Approximate time required:** 45 minutes / 1 class period

#### **Content and Instructional Strategies:**

1. Distribute analog clock models to every student.
2. Using an analog clock model displayed on the document camera, review each of the elements of "Telling Time with an Analog Clock: Reference Sheet."
  - c. Review the parts of an analog clock. Elicit student input for each analog clock part and instruct students to match their individual clocks to the document camera model of each of the following:
    - i. larger hand: represents minutes
    - ii. shorter hand: represents the hour

- iii. faster, thinner hand: represents seconds
    - iv. each tick mark: represents one minute
    - v. 5 tick marks: span distance between numerals
  - d. Using an analog clock model displayed on the document camera, review special vocabulary of telling time, eliciting student response on:
    - i. 60 minutes in each hour
    - ii. 60 seconds in each minute
    - iii. “3” represents 15 minutes or “quarter past” the hour
    - iv. “6” represents 30 minutes or the half hour
    - v. “9” represents 45 minutes or “quarter to” the hour
  - e. Using an analog clock model displayed on the document camera, lead students in counting the five-minute intervals around the clock face.
  - f. Ask students to show 25 minutes on the clock and to name the corresponding numeral on the clock face.
  - g. Ask students to show 35 minutes on the clock and to name the corresponding numeral on the clock face.
- 3. Practice: Divide students into pairs and direct them to work together to consider the following schedule change.
  - a. Model calculation of a new start time by asking: “If class is scheduled to begin at 7:15 but now must start five minutes earlier, what is the new start time?”
    - i. After consulting with partner, students write down answer and move clock hands to represent this time.
    - ii. Ask students for solution. Count out time change by touching the clock in 5-minute increments and moving the clock hands.
    - iii. Write new start time “7:10” and draw corresponding clock face on paper displayed on document camera.
  - b. Model calculation of the new end time by asking:” If the class lasts 40 minutes, what time will it end?”
    - i. After consulting with partner, students write down answer and move clock hands to represent this time.
    - ii. Ask students for solution. Count out time change by touching the clock in 5-minute increments and moving the clock hands.
    - iii. Write new end time “7:50” and draw corresponding clock face on paper displayed on document camera.
  - c. Model additional example.
    - i. “If a 6:35 class begins 10 minutes later, when is the new start time?”
    - ii. “If the class is scheduled to last 55 minutes, what is the new end time?”
    - iii. After consulting with partner, students write down answer and move clock hands to represent this time.
    - iv. Ask students for solutions. Count out time change by touching the clock in 5-minute increments and moving the clock hands.
    - v. Write answers (6:45 start time and 7:40 end time) and draw corresponding clock faces on paper displayed on document camera.

4. Display on the document camera: “Telling Time with an Analog Clock.” Tell students that this reference will remain visible throughout the lesson and instruct them to refer to it while they work.
5. Distribute handouts to every student and describe the lesson’s scenario: snow has resulted in a one-hour late opening for school. The daily class schedule must be changed to allow each subject to be studied in its regular rotation but for less than its usual time. Student schedules will be compiled into a book of proposals that will be forwarded to the school principal. Students may work in pairs but each student must complete individual handouts. Students may freely create a new schedule with the restrictions that:
  - a. all classes must meet for at least 5 minutes during the day
  - b. school starts at 10:30
  - c. school ends at 3:40
6. In twenty minutes, after student pairs have made progress on their summary schedules and clock faces, refocus class for whole group discussion.
  - a. Student pairs share revised schedules for the first period (science/social studies) of the day.
  - b. Teacher displays on the document camera the new start and end times, duration, and analog clocks for these different options.
  - c. Teacher clarifies student questions and corrects errors as necessary.
7. Students continue to work in pairs to complete the handouts. If time runs out, handouts can be finished for homework.
8. Extension: During the next lesson, students finish and decorate their final schedules for inclusion in a book that will be assembled, titled “Proposed Revised Class Schedules for a One Hour Late Opening,” and delivered to the principal for his consideration. As a Language Arts lesson, students will write persuasive letters promoting the benefits of their particular class schedule.

**Evaluation/Assessment:**

Formative: Teacher monitors student participation and accuracy of responses during class discussions and while circulating as students are working in pairs.

Summative: On an exit card (see “Calculating Time” attachment), students are asked to calculate a new start and end time for a class and to represent these times on an analog clock.

**Differentiation and Adaptations:**

- Special education students can be paired with general education students for support.
- For greater visual discrimination, colored pencils or markers can be used for drawing the clock hands (e.g., red for hour hand, green for minute hand).
- Larger clocks can be provided for students challenged by fine motor skills.

**References:**

Florida Center for Instructional Technology, College of Education, University of Southern Florida. (2010). *Clipart ETC*. Retrieved October 20, 2010 from <http://etc.usf.edu/clipart/>

## Telling Time with an Analog Clock: Reference Sheet

### Parts of an Analog Clock

Larger hand	Minutes
Shorter hand	Hour
Faster, thinner hand	Seconds
1 tick mark	1 minute
5 tick marks	5 minutes
5 tick marks	distance between every two numerals

### Time Measurements

60 seconds	= 1 minute
60 minutes	= hour

### Reading a Clock

Numeral	Minutes	Special Vocabulary	Written Form / Digital Clock	<u>Analog Clock</u>
3	15	quarter past the hour	__: 15	 4:15
6	30	half hour	__: 30	 9:30
9	45	quarter to the hour	__: 45	 1:45
5	25	--	__: 25	 5:35
7	35	--	__: 35	 5:35

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Number: \_\_\_\_\_

**Summary: New Timetable for Classes**

**Directions:** School opens one hour late due to snow. Use the following summary sheet to create a new class timetable. Each class must meet for at least five minutes. School begins at 10:30 and ends at 3:40.

<b>Class</b>	<b>Original Start Time</b>	<b>Original End Time</b>	<b>Original Duration</b>	<b>New Duration</b>	<b>New Start Time</b>	<b>New End Time</b>
<b>Science/SS</b>	9:30	10:15	45 min.		10:30	
<b>Centers</b>	10:15	11:05	50 min.			
<b>Snack</b>	11:05	11:15	10 min.			
<b>Reading</b>	11:15	12:30	1 hr. and 15 min.			
<b>Lunch / Choice</b>	12:30	1:15	45 min.			
<b>Math</b>	1:15	2:30	1 hr. and 15 min.			
<b>Recess</b>	2:30	2:50	20 min.			
<b>Language Arts</b>	2:55	3:40	45 min.			3:40

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Number: \_\_\_\_\_

**Summary: Clock Faces for New Class Schedule**

**Directions:** Using the schedule you created on the summary sheet,  
1) write the new start and end times for each class next to each clock. and then 2)  
draw hands on each clock to represent this time.

Science / SS:	Start Time =		End Time =	
Centers:	Start Time =		End Time =	
Snack:	Start Time =		End Time =	
Reading:	Start Time =		End Time =	
Lunch/Choice:	Start Time =		End Time =	
Math:	Start Time =		End Time =	
Recess:	Start Time =		End Time =	
Language Arts:	Start Time =		End Time =	

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Number: \_\_\_\_\_

## Calculating Time

Lunch is usually scheduled to begin at 12:30. On a snow day, lunch begins 15 minutes earlier than usual. What time will lunch begin? Draw hands on the clock to represent this time.



Start Time \_\_\_\_\_

If lunch lasts 25 minutes, what time will lunch end on this snow day? Draw hands on the clock to represent this time.



End Time \_\_\_\_\_

Class starts at the time shown on this clock.  
What time does this class start? \_\_\_\_\_



Class ends at the time shown on this clock.  
What time does this class end? \_\_\_\_\_



How long does this class last? \_\_\_\_\_

## **Problem #4**

### **Thoughts:**

*I am a keen advocate of nurturing a global perspective in my students and integrating an international worldview into every nook and cranny of the curriculum. This problem appealed to me because it provides a strong link to map and globe skills and advances thinking beyond the students' immediate boundaries of their county or town. I also liked the proportional aspect of the problem. Students can take a distance to which they easily relate and expand this knowledge to the country and even the world. When my daughter's 4<sup>th</sup> grade New York City class tackled a similar problem during an immigration unit, her teacher asked the students to make calculations to each family's place of ancestry. She took the students on a 20-block (1 mile) walk as a reference. Our family used this walk uptown as the basis for a calculation of the distance to Italy.*

### **Topics**

Measurement (SOL 3.16, 5.12) and Computation and Estimation (SOL 4.7, 5.3)

### **Expected Heuristics**

Solve a simpler problem  
Choose the operation

### **Appropriate Grades**

Grades 3 through 5.

### **Problem**

- ***Where In the World?*** Think of a place in North America that you would like to visit. What if you had to walk to get there? How long would it take? Research to find out the number of miles to your chosen location. Use a stopwatch to find out how long it would take you to walk a mile. Use that amount of time to make an estimate.

### **Solution**

Depends on student selected destination.

### **Methods/ Discussion**

- Problem can be connected to student interest, recent travel, or topics of study.
- Example: Student selects Grand Canyon National Park as a destination and uses Googlemaps to determine that it is 2,263 miles from Matoaka Elementary School. Using "The Cardinal Way", a one-mile nature loop at Matoaka, the student determines that it takes 28 minutes to walk 1 mile. The student then estimates that in an hour, he/she could travel approximately 2 miles and therefore a walking trip to the Grand Canyon would require approximately  $2263 / 2$  or 1131.5 hours.
- The problem could be extended from estimation to calculation. The student calculates that walking to the Grand Canyon National Park will require  $28 \times 2263$  minutes = 63,364

minutes or 1056 hours and 4 minutes. If a student walked 12 hours every day, he/she would reach the Grand Canyon in 88 days or approximately 3 months.

**Weekly Problem: Elementary School (2003).** *Math by the Month, Teaching Children Mathematics*. Retrieved September 9, 2010, from <http://www.nctm.org/resources/archive.aspx?id=3604&journalid=4>

### **Problem #5**

#### **Thoughts:**

*I have revised this problem from its original submission to eliminate the teacher introduction of a chart. In so doing, my aim is to remove the modeling that shifted this problem into an exercise.*

*I specifically chose this problem to teach in my practicum classroom as a supplement to four fact families. However, in the end I opted for another approach to compliment my teacher's request that I focus on math mountains as an extension of her instruction.*

**Topics:** Patterns, Functions, and Algebra (SOL 3.24)

#### **Expected Heuristics**

- Find a pattern.
- Make a table.

**Appropriate Grade:** Grade 3

#### **Problem**

- **How Many Ways?** How many addition number sentences can you write that equal a certain number? For example, if your number is 3, you can write four number sentences:
  - $0 + 3 = 3$
  - $3 + 0 = 3$
  - $1 + 2 = 3$
  - $2 + 1 = 3$

Try this activity with other numbers. What do you discover?

#### **Solution**

1 = two addition number sentences, 2 = three number sentences, 3 = four number sentences, 4 = five number sentences. The total possible combination of number sentences for number  $x$  equals  $x + 1$ .

#### **Methods/ Discussion**

- Model the procedure to find the total addition number sentences for 3 and 8. Solicit student input as you proceed. Total the number of sentences and record this total below the column such that the relationship (between 3 and 4, 8 and 9) is not immediately obvious.

- Divide students into small groups of four and assign each group member two numbers below 12. Students are encouraged to work independently to determine the number of addition sentences for their numbers and then to collaborate to determine if there is a pattern. Small groups share their results with the class.
- Task children with testing conclusions by investigating higher numbers. Students are given numbers between 11 and 30 (based on math confidence and ability) and are encouraged to collaborate and discuss as they determine the correct number of addition sentences.

**Weekly Problem: Elementary School (2001). *Math by the Month, Teaching Children Mathematics*. Retrieved September 25, 2010, from <http://www.nctm.org/resources/archive.aspx?id=3604&journalid=4>**

### **Problem #6**

#### **Thoughts:**

*I am attracted to mapping activities due to their connection with social studies and authentic application to the real world. This problem offers an additional challenge in that students would use different non-standard measuring tools of their choice. Measurements would be compared and contrasted at the end of the activity in a whole group chart.*

**Topics:** Measurement (SOL 3.14), Computation and Estimation (SOL 3.12), and Geography (SOL 3.6)

**Expected Heuristics:** Use equations and formulas; Make an organized list

**Appropriate Grade:** Grade 3

**Problem: *How Big is Your Space?*** Working in a small group, draw a scale map of the classroom perimeter using one of a selection of different measuring instruments: ruler of US customary units, metric ruler, and non-standard measuring tools (e.g., composition notebook and pencil). Measure the perimeter of the room noting the measurements of doors, windows, and cubbies. Draw your map on graph paper and create a compass rose, a key for your scale drawing, and a map legend with symbols for windows, doors, and cubbies. Label the map with the total perimeter measurement of your classroom (e.g., The Perimeter of Our Classroom = 1582 crayons)

**Solution:** Depends on classroom dimensions and measurement tool used.

#### **Methods/ Discussion**

- Problem can be connected to Social Studies lesson on Map Skills.
- Teacher divides students into small groups that will each measure the room with a different measuring tool. Dividing into 4 groups will permit measurement to begin at four corners of classroom thus minimizing crowding and allows for use of:
  - ruler or yardstick of US customary units

- metric ruler or meter stick
- at least 2 different non-standard measuring tools (e.g., composition notebook and pencil)
- Students work together to measure, record, aggregate, and double-check data.
- Students collaborate to draw map to scale and to create accurate map tools:
  - compass rose noting four cardinal directions
  - key for map scale
  - map legend noting symbols for doors, windows, and cubbies.
- Students compare perimeter measurements and discuss how fixed dimensions of the classroom can be expressed differently depending on the measuring tool used.

Adapted from Weekly Problem: Elementary School (2008). *Math by the Month, Teaching Children Mathematics*. Retrieved October 12, 2010, from <http://www.nctm.org/resources/archive.aspx?id=3604&journalid=4>

Additional resource: Measuring Up With Apples (1998). *Ask ERIC Lesson Plans*. Retrieved October 12, 2010, from <http://www.theteachersguide.com/lesson%20plans/Math/MEA0014.html>

### **Problem #7**

#### **Thoughts:**

*The use of real receipts and the authentic scenario in the familiar environs of a supermarket create a strong appeal for this problem. It is based on a favorite problem from my son's 3<sup>rd</sup> grade math class. As a result of this activity, he became highly engaged in our trips to the market, estimating total costs - just in case.*

*I have opted for partner work for this and other problems because it requires students to externalize their thoughts and explain their thinking. As a result, students are able to problem solve and revise as they proceed through a problem and are able to support and enhance each other's work.*

**Topics:** Number and Number Sense (SOL 3.7), Computation and Estimation (SOL 3.8 and 3.12), Health Knowledge and Skills (SOL 3.1a), and Economics (SOL 3.9)

**Expected Heuristics:** Use equations and formulas; Make an organized list

**Appropriate Grade:** Grade 3

**Problem: *Oops, Where's My Wallet?*** A father shops at the supermarket and after the cashier has added up all his purchases at the register, he discovers that he has forgotten his wallet at home. Fortunately, he has a single \$20 bill in his pocket. Working in pairs, students edit down the total cost of a grocery purchase to below \$20 while still including enough food for a day's meals for a family of four. Students use authentic grocery receipts collected from the community; identifying information has been excised and receipts have been checked for appropriateness (alcohol, tobacco products, and other sensitive items have been blacked out).

**Solution:** Depends on grocery receipt information.

### Methods/ Discussion

- Problem can be connected to health lesson on nutrition and social studies lesson on economics.
- Teacher divides students into pairs and distributes a grocery receipt to each pair. Teacher instructs students to consider maximizing the amount of food that can be purchased for \$20 and to distribute food choices among a variety of products. For example, purchasing 5 boxes of cereal for \$3.99 each satisfies the \$20 limit but does not address need to balance food choices.
- Students work together to determine a selection of groceries representing a balance of food that totals \$20 or less. In order to practice adding decimals as well as three-digit and four-digit addition, students will use pencil and paper to make calculations rather than using calculators.
- After completing their calculations, students prepare a large-format summary for class display:

What we bought for \$20

Quantity	Description of food product	Cost
Total number of items		TOTAL COST

- In whole group discussion, student pairs present food lists and compare different options.

Adapted from problem remembered from my children's elementary school math instruction. Additional resource: International Reading Association (2010). *Authentic Writing Experiences and Math Problem-Solving Using Shopping Lists*. Retrieved October 19, 2010, from <http://www.readwritethink.org/resources/resource-print.html?id=298&tab=3>

### **Problem #8**

#### **Thoughts:**

*I like the multiple levels of challenge posed by this problem, another adapted from my children's elementary math experience. Students must first represent their own name, confer with their classmates to select one name from the group, and then work together to try to match the trays to all of the collaborative groups. Students will calculate multiple times during this activity and will discuss their totals and conclusions with their group members.*

**Topics:** Number and Number Sense (SOL 1.2, 3.1)  
Computation and Estimation (SOL 2.7, 3.8, 4.5)

**Expected Heuristics**

- Guess and check
- Make and organized list

**Appropriate Grades:** Grades 1 through 4

**Problem: The “Name that Name” Game:** In small groups, students count the number of letters in their individual names. They select one name to represent with base ten blocks. They then create a base ten representation of the number of their group’s total name letters. Later, in their small groups, students examine each group’s base ten models and assign individual and group names to all base ten models (including their own).

**Solution:** Depends on student names

**Methods/ Discussion**

- In order to maintain secrecy, manipulatives must be identical between groups.
- Depending on grade level, students use first name, first and last names, or first, middle, and last names.
- Students are divided into small groups. On whiteboard, teacher lists group numbers and corresponding student names.
- In small groups, students work individually with base ten manipulatives (unifix cubes, c- rods, links work well) to calculate and represent the number of letters in each of their names. The group then selects one of these representations, labels with student name, places on tray, and covers with paper to maintain secrecy.
- Group reconstructs the selected name from new blocks. Group works together to combine all group names in base ten blocks, then places these blocks on the tray, and re-covers with paper. Teacher removes tray.
- When all groups have completed activity, students gather on one side of classroom with backs turned to teacher. Teacher assigns random label (e.g., color, animal, country) to each tray and randomly places trays around classroom.
- Small groups work together to examine trays and to determine which tray belongs to which group and what individual name is represented. Students work quietly in order to not alert rival groups.
- After all groups are finished, students share results and teacher reveals solutions.

Adapted from problem remembered from my children’s elementary school math instruction.

**Additional resource:** Trafton, P.R. & Thiessen, D. (1999). *Learning through problems: Number sense and computational strategies*. Portsmouth, N.H.: Heinemann.

## Web Resources

### **Thoughts:**

*Although my practicum math class incorporates IXL Math into its regular small group rotation, the web resources assignment and information from our Technology class has expanded my awareness of other online options to scaffold classroom math instruction. I selected the United Nations and Weather websites with the aim of linking math to social studies and science instruction. The Math Playground site is an alternative to IXL and replicates many familiar board game templates.*

## United Nations Statistics Division

**Address:** <http://unstats.un.org/unsd/demographic/>

### **Description of contents:**

For a global perspective of international populations and living conditions, the United Nations Statistics Division offers a broad range of global data on “demographic and social concerns.” The site is easy to navigate and draws from individual country databases as well as from UN sources. Separate pages offer comprehensive data across ten broad categories of demographic information. Every page details the standards and methods for individual databases. Data is presented in pdf files as well as html and excel format from sources released as late as September 2010. This site provides a useful math tie-in for social studies lessons, providing comprehensive community and economic information for countries around the world.

Databases include:

- Size and structure of population, births, deaths, migration, density, urbanization, and ethnocultural characteristics:
- Family formation, families and households: Data on marriage and divorce rates by urban/rural residence, age of bride and groom, and birth rates current as of 2005/2006.
- Health, human functioning and disability
- Housing and its environment
- Learning and education
- Economic activity
- Allocation of time and time use (recreation and leisure time) (incomplete data)
- Social security and welfare (indirect link)
- Distribution of income and consumption; wealth and poverty (indirect link)
- Public order and safety, offenders and their victims (indirect link)

### **How you can use this source:**

- Analyze demographic data (population, birth rate, literacy, ethnocultural breakdown) for an individual country across time
- Compare and contrast population data using graphs, charts, maps

- Between countries on different continents (Europe vs Africa)
- Between countries within a continent (Algeria vs Chad)
- For a specific nation across decades (1950 vs 2000)
- Compare analysis from UN data with interactive cartograms such as <http://show.mappingworlds.com/world/>

**United Nations Statistics Division, (2010). Retrieved September 9, 2010 from <http://unstats.un.org/unsd/demographic>**

## **Math Playground**

**Address:** <http://www.mathplayground.com/games.html>

### **Description of contents:**

For students who enjoy electronic gaming, Math Playground offers interactive math activities for all grade levels broken down into three broad categories: Math Games, Word Problems, and Logic Puzzles. The site also provides student support through a page of “Math Videos” with more than seventy short presentations addressing numbers and their properties, arithmetic, fractions, decimals, percent, ratio and proportion, measurement and data, algebra and pre-algebra, and geometry.

The Math Games page includes practice of math concepts from general math knowledge to algebraic reasoning across two dozen games including titles such as “Math Millionaire,” “Calculator Chaos,” and “Bridge Builders.” The Logic Puzzle page offers a recognizable collection of games such as Connect 4, Tangrams, Tetra Squares, and Battleship. The Word Problem page provides grade level tags on its ten categories of activities and presents games with sports themes, puzzles, and SAT format simulations. An example of Math Playground is Grand Slam Math on the Word Problem page. This baseball themed game has two levels and provides practice for both single and multistep word problems. The first level has 15 addition and subtraction problems and the second has 12 multiplication and division problems. The introduction alerts students that the game is designed to become progressively more difficult. When the student answers incorrectly, an explanation of how to provide the solution is offered but the answer is not provided.

### **How you can use this source:**

- Students can use this site independently to review and reinforce math concepts.
- Teachers can use this site as a computer center for review and reinforcement. Given the large selection of options and levels, teachers can find an appropriate level of challenge to meet each student’s unique abilities.

Math Playground, (2010). Retrieved September 9, 2010 from <http://www.mathplayground.com/games.html>

## **Wonderful World of Weather**

**Address:** <http://www.proteacher.com/cgi-bin/outside/site.cgi?id=11486&external=http://www.k12science.org/curriculum/weatherproj/index.html&original=http://www.proteacher.com/110022.shtml&title=Wonderful%20World%20of%20Weather>

### **Description of contents:**

The Stevens Institute of Technology's Center for Innovation in Engineering and Science Education designed this interdisciplinary site as a resource tool for elementary school teachers. Wonderful World of Weather is one of Stevens' nine "Real Time Data Projects" linked to curriculum standards and is the only module aimed for grades K-6. Seven of the eight remaining Real Time Data Projects are appropriate for grades five or six and focus on activities such as ocean waves and tides, seismic activity, cargo ship navigation, and ozone levels. The ninth module addresses aircraft flight data and is intended for high school students. The Wonderful World of Weather module allows students the opportunity to apply math skills to their lived experience of weather. The Teacher Guide section is organized into three pages: Using Real-Time Data, Implementation Assistance, and Lesson Plans. The Lesson Plan page provides a complete learning unit comprised of three introductory activities, ten follow-up core lessons, handouts, and sample handouts completed by students. Through this site, the teacher has the flexibility to tailor the weather investigation to the students' local community or to expand their perspective to a national or global view. All lesson plans are grounded in the application of math concepts. In addition, the site lists "Ask An Expert" and Stevens Institute "Project Leader" contact information for this weather module.

### **How you can use this source:**

- "Reference Page" provides links to more than twenty-five weather-data sites including one that compiled Martian weather data collected from the Mars Global Surveyor.
- Core Lesson Plan #1: "Relative Weather" investigates weather conditions in discrete locales and focuses on math by asking students to compile and organize data and then to calculate averages and display results in graphs and charts.
- Core Lesson Plan #5: Elevation and Temperature asks students to calculate differences in data readings for cities at the same latitude but different elevations.

Stevens Institute of Technology (2010). Wonderful World of Weather. Retrieved September 10, 2010 from <http://www.proteacher.com/cgi-bin/outside/site.cgi?id=11486&external=http://www.k12science.org/curriculum/weatherproj/index.html&original=http://www.proteacher.com/110022.shtml&title=Wonderful%20World%20of%20Weather>

## Conference Review

### **Thoughts:**

*Both of the Math Day Conference sessions introduced me to completely new concepts. I had absolutely no familiarity with patty paper as a math tool and while pattern blocks are a standard manipulative in my preschool I had never made any connection between them and fractions. The pattern block work made me realize how ingrained algorithms are in my adult brain. I understand and am completely comfortable with these algorithms and found that the unpacking of these well-worn routines into the foundational meaning was very challenging. In some ways, this experience felt like learning a new language or an advanced yoga pose. Difficult but ultimately worthwhile.*

### **Patty Paper Profusion: Mathematical Uses for Patty Paper, Grades 2-6**

Patty paper is a low-cost, durable manipulative sourced from the restaurant industry that can be used on the paper and pencil version of the SOLs. Measuring 5.25 inches per side, these translucent perfect squares are effective problem-solving tools for elementary students, particularly for visual learners. Using patty paper as an instructional aid, students can write on the paper itself, easily erase pencil marks, see through the translucent paper to trace shapes and objects underneath, and create visible creases. In addition, the right angle at each corner of the patty paper provides students a quick means to determine if an angle is right, obtuse, or acute.

With repetition and practice, patty paper work scaffolds students' skill building and ability to visualize geometric solutions. More importantly, when using patty paper, students are forced to slow down their work process thus reducing errors by increasing thinking time and visualization of strategies. Using patty paper can also increase student focus by blocking extraneous information printed on the page and emphasizing key attributes of a problem. For example, tracing a particular shape on patty paper can eliminate the distractions of superimposed or overlapping shapes and focus student attention on the one shape being analyzed.

Patty paper must be introduced as a tool to supplement specific problems and used systematically in meaningful contexts. For geometry instruction, patty paper applications include transformations; attributes of plane and solid figures; symmetry; congruence; similarity; points, lines, line segments, rays, and angles; perpendicularity, parallelism, and intersection; and coordinate geometry. In addition to the uses detailed above, patty paper can be used to solve geometry problems by tracing and then reorienting geometric figures into a more familiar and recognizable position, positioning the paper corner to determine perpendicularity, and using as a straight edge to extend lines thus facilitating the discrimination of parallel lines.

This workshop also modeled three different instructional strategies for use in any content area. The "round robin" written brainstorming strategy is an effective tool for scaffolding different abilities within a heterogeneous group. While all students have equal opportunity to contribute to the collaborative brainstorming, greater challenge is encountered at the end of a rotation after obvious or easier inputs have been exhausted. Assigning struggling students to the initial positions within a round robin allows them to contribute equally with the more accomplished students positioned at the end of the rotation. Second, a group graphic organizer was introduced in tandem with a "Think/Pair/Share" analysis of a set of standards. After each student pair made notes in a designated section of a shared document, the larger group edited and synthesized these inputs to create a composite summary of the document being reviewed. Lastly, the presenter modeled a questioning strategy that showed no discernible order thus forcing

listeners to be attentive to the comments of fellow students and prepared to answer if called upon next.

In all, this workshop was highly valuable in presenting a thorough explanation of how to integrate a simple, low-cost instructional aid into geometry instruction. The presenter supplemented her oral presentation with a comprehensive seventeen-page handout of detailed examples from released SOLs. She systematically went through the handout offering detailed examples of patty paper use. In addition, the presenter seamlessly and effectively incorporated three examples of pedagogy into her content discussion. I envision using these techniques as well as the patty paper in my future teaching. Given the myriad types of learners, a teacher must continuously strive to present a broad array of learning tools as options for student use. Patty paper is a low-risk, easily integrated teaching tool that may open a window of understanding for visual learners or students who need to reduce distractions in written problems. I will certainly add it to my teaching tool inventory.

**Relevant SOLs include:**

1.16 The student will draw, describe, and sort plane geometric figures (triangle, square, rectangle, and circle) according to number of sides, corners, and square corners.

2.21 The student will identify and create figures, symmetric along a line, using various concrete materials.

3.18 The student will analyze two-dimensional (plane) ~~and three-dimensional (solid)~~ geometric figures ~~(circle, square, rectangle, triangle, cube, rectangular solid [prism], square pyramid, sphere, cone, and cylinder)~~ and identify relevant properties, including the number of corners, square corners, edges, and the number and shape of faces, using concrete models.

3.20 The student, given appropriate drawings or models, will identify and describe congruent and symmetrical, two-dimensional (plane) figures, using tracing procedures.

4.14 The student will investigate and describe the relationships between and among points, lines, line segments, and rays.

4.18 The student will identify the ordered pair for a point and locate the point for an ordered pair in the first quadrant of a coordinate plane.

5.14 The student will classify angles and triangles as right, acute, or obtuse.

6.14 The student will identify, classify, and describe the characteristics of plane figures, describing their similarities, differences, and defining properties.

6.15 The student will determine congruence of segments, angles, and polygons by direct comparison, given their attributes. Examples of noncongruent and congruent figures will be included.

## A Hands-On Approach to Multiplying and Dividing Fractions: Pattern Blocks

Pattern blocks are a manipulative that can be used to teach an area model of fractions. While pattern blocks are often used to study geometry, in this instance, the blocks are used to develop students' cognitive understanding of fractions. Fractions are more difficult for students to grasp because they are variable rather than fixed. That is, they express a relationship to a whole number and their value depends upon the value of the whole number to which they are measured. Fractions represent a flexible relationship and when teaching fractions to students, the flexible nature of fractions must be nurtured and reinforced. Pattern blocks are a concrete way to nurture this flexible perspective and to demonstrate relationship of fractions relative to whole numbers. In using pattern blocks for fraction instruction, teachers must be careful not to create confusion by using geometry vocabulary. Instead, teachers should exclusively focus on fractions by using color names to describe pattern blocks rather than using their shape names (such as hexagon and rhombus).

The certainty of the pattern blocks and their fixed interrelationships add a finite and concrete dimension to the inherently amorphous nature of fractions. For example, in pattern blocks, two red trapezoids always equal one yellow hexagon and six green triangles always equal one yellow hexagon. These correlations are fixed and applying fractions to these static relationships adds certainty to fraction instruction. In addition, the hands-on nature of pattern blocks supports learning for students who benefit from concrete manipulation of objects.

This presentation provided in-depth modeling of teaching geometry with pattern blocks. Through six full pages of activities, we explored the multiple permutations of pattern blocks that equated with one whole yellow hexagon, the fractional relationships between four sizes of pattern blocks, representations of multiplication and division of fractions using pattern blocks, and applying these multiplication and division fraction number sentences to word problems.

Throughout the presentation, the importance of wording and the specific meaning of fractions were emphasized. For example, teachers can use pattern blocks to underscore the explicit meaning of fractions by showing that a blue rhombus is one-third because three of these pattern blocks comprise one yellow hexagon or one whole. When one whole changes and is represented by two yellow hexagons, then students can see that six blue rhombi equate to this new whole and therefore equal one-sixth of the two yellow hexagons. This hands-on method translates abstract language into concrete, observable, and tangible examples. The rhombi can be manipulated to cover the hexagon.

Pattern blocks clarify why multiplying fractions results in a smaller number. As students manipulate pattern blocks, it becomes clear that multiplication of fractions means that you are taking a piece of a piece. For instance, in representing " $1/2 \times 1/3$ " with pattern blocks, one half of a blue rhombus ( $1/3$  of whole yellow) is clearly visible as one small green triangle or  $1/6$  of the whole (the yellow hexagon). Using manipulatives demonstrates how and why the solution becomes smaller.

Language also was stressed in explaining how students determine what is the first number or fraction to which the fractional relationship is being expressed. For example, in division the question being asked in " $1/2 \div 1/6$ " is "how many  $1/6$ ths fit into  $1/2$ ". As teachers emphasize the order of the fractions and the language to express this relationship, they underscore the meaning by manipulating the small green triangles ( $1/6$ ) to fit into a red trapezoid ( $1/2$ ). Using pattern blocks gives meaning to the language of fractions. Students can use the comprehensible examples of these simple blocks to absorb the meaning of multiplication and division of fractions. Once these relationships and terms are mastered, students apply this strong foundational understanding to more complex fractions.

As an adult who is comfortable with my fraction skills, I found pattern blocks to be illustrative and exciting. I will definitely incorporate this methodology in my teaching. The students in my practicum seem to respond to hands-on demonstrations and manipulatives and those students with the weakest skills are reluctant to attempt any math without step-by-step teacher modeling. I believe that using pattern blocks and encouraging collaboration and discussion would foster student understanding and a strong basis on which more complex fraction instruction could be built.

**Relevant SOLs include:**

1.6 The student will identify and represent the concepts of one-half and one-fourth, using appropriate materials or a drawing.

2.4 The student will identify the part of a set and/or region that represents fractions for one-half, one-third, one-fourth, one-eighth, and one-tenth and write the corresponding fraction.

3.5 The student will a) divide regions and sets to represent a fraction; and b) name and write the fractions represented by a given model (area/region, length/measurement, and set). Fractions (including mixed numbers) will include halves, thirds, fourths, eighths, and tenths.

4.2 The student will a) identify, model, and compare rational numbers (fractions and mixed numbers), using concrete objects and pictures; b) represent equivalent fractions; and c) relate fractions to decimals, using concrete objects.

5.2 The student will a) recognize and name commonly used fractions (halves, fourths, fifths, eighths, and tenths) in their equivalent decimal form and vice versa;